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WOW!

Free Coloring Algorithms!!
or
Restricted Coloring Problems on Restricted Classes of Graphs

FREE!

HEY!

Restricted Coloring Problems

- *Distance-1* (or *proper*) coloring: adjacent vertices receive distinct colors
- *Acyclic* coloring: proper coloring with no 2-colored cycle
- *Star* coloring: proper coloring with no 2-colored P_4

$$\chi(G) \leq \chi_a(G) \leq \chi_s(G) \quad \text{and} \quad \Phi(G) \supseteq \Phi_a(G) \supseteq \Phi_s(G)$$

How can we use algorithms for one coloring problem to solve another?

Theorem (Gebremedhin et. al., 2008)**.** *If G is a chordal graph, then every proper coloring of G is also an acyclic coloring.*

Chordal graphs can be colored in linear time ⇒ FREE algorithm for acyclic coloring on chordal graphs!

When are Two Problems Equivalent?

Distance-1 ⇔ Acyclic

A graph G is *even-hole-free* if it does not contain an induced cycle with an even number of vertices.

Theorem. *A graph G is even-hole-free if and only if every distance-1 coloring of G is also an acyclic coloring.*

Corollary. *If G is an even-hole-free graph, then $\chi(G) = \chi_a(G)$.*

Corollary. *Any algorithm for finding an optimal coloring of an even-hole-free graph will also find an optimal acyclic coloring.*

Distance-1 ⇔ Star

A graph is *trivially perfect* if it does not contain C_4 or P_4 as an induced subgraph.

Theorem. *A graph G is* trivially perfect *if and only if every distance-1 coloring of G is also a star coloring.*

Corollary. *If G is a trivially perfect graph, then $\chi(G) = \chi_a(G) = \chi_s(G)$.*

Acyclic ⇔ Star

A graph is a cograph if it does not contain P_4 as an induced subgraph.

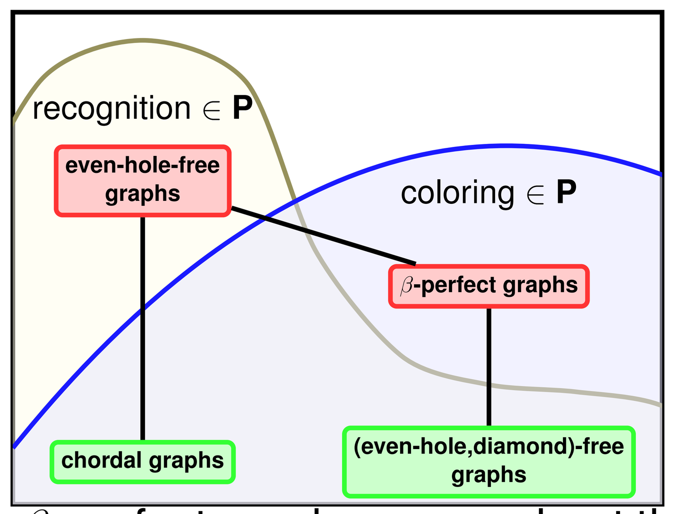
Theorem. *A graph G is a cograph if and only if every acyclic coloring of G is also a star coloring.*

Corollary. *If G is a cograph, then $\chi_a(G) = \chi_s(G)$.*

Algorithms

Acyclic coloring

If we know our graph is even-hole-free, we can simply run any algorithm/heuristic for proper coloring. However, it is not currently known whether even-hole-free graphs can be colored optimally in polynomial time.



β-perfect graphs are a subset that can be colored in polynomial time, but no efficient recognition algorithm is known!

The class in the following theorem is a proper subclass of the β-perfect graphs.

Theorem. *There exists a linear-time algorithm for finding an optimal acyclic coloring of (even-hole,diamond)-free graphs.*

However, even-hole-free graphs are (currently) costly to recognize ($O(n^{15})$). But we may be able to avoid recognition.

Problem: Find an efficient robust algorithm for coloring even-hole-free graphs.

Star coloring

Theorem. *There exists a linear-time algorithm for finding an optimal star coloring of a cograph G.*

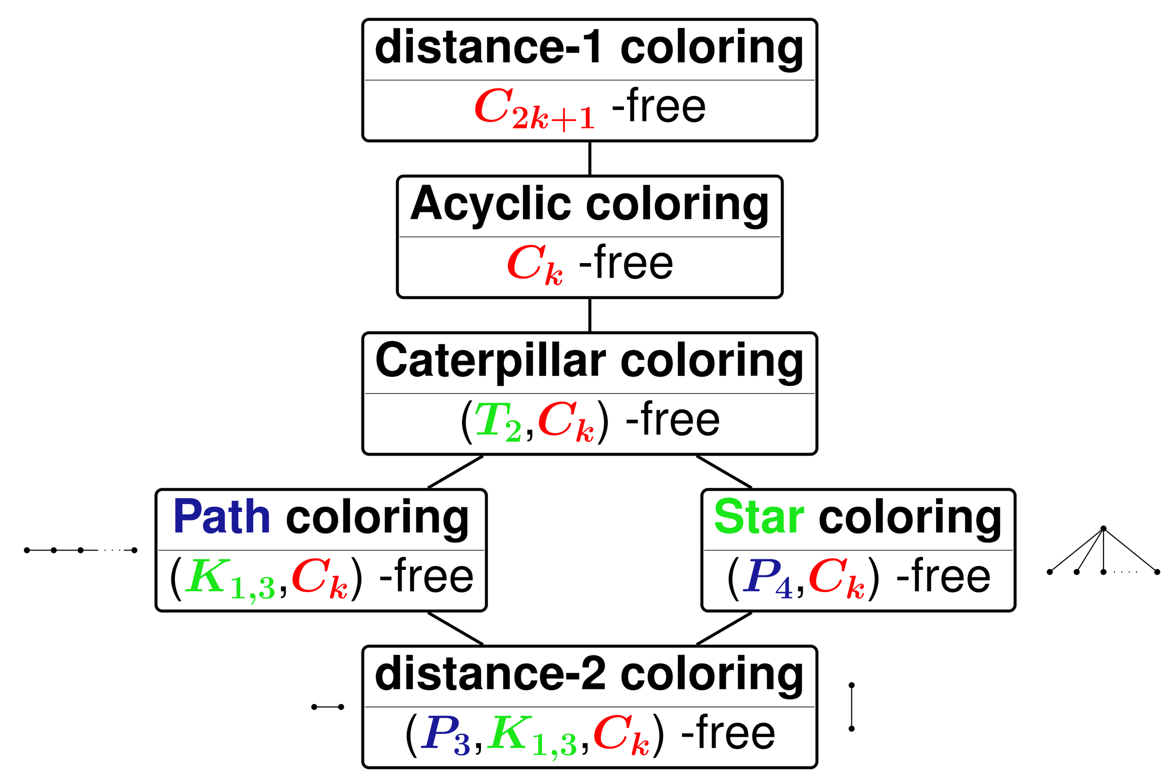
What about the complexity on other classes? The split graphs are an interesting case:

Problem: Determine the complexity of star coloring on split graphs.

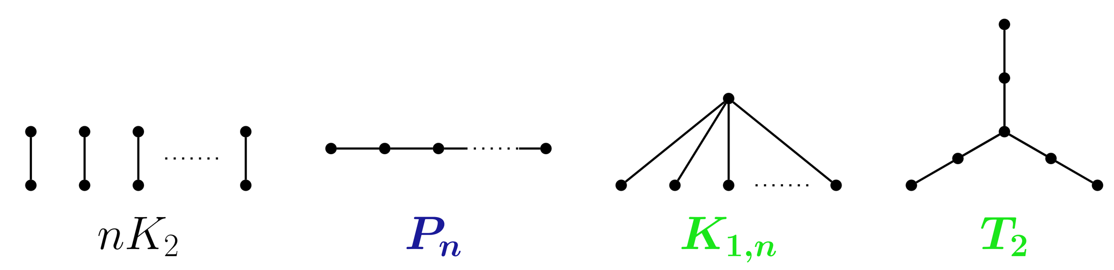
Unifying Concept: Forbidden Subgraphs

Generalizing to other restricted coloring problems:

- *Distance-2* coloring: every 2-colored induced subgraph is a matching.
- *Caterpillar* coloring: every 2-colored induced subgraph is a disjoint collection of caterpillars.
- *Path* (or *linear*) coloring: every 2-colored subgraph is a disjoint collection of paths.
- etc...



Problem	2-Colored Induced Subgraphs
proper coloring	C_{2k+1}-free
acyclic coloring	C_k-free
caterpillar coloring	(T_2,C_k)-free
Star coloring	(P_3,C_k)-free
Path coloring	($K_{1,3}$,C_k)-free
distance-2 coloring	(P_2,$K_{1,2}$,C_k)-free



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